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**A STUDY ON THE SPATIAL-TEMPORAL DYNAMICS OF WORMHOLES IN A  
BRANEWORLD MODEL**

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# **A STUDY ON THE SPATIAL-TEMPORAL DYNAMICS OF WORMHOLES IN A BRANEWORLD MODEL**

## **ABSTRACT**

The dynamic wormhole models that were previously introduced focused on the dynamics of the wormhole itself of either rotating or evolutionary in characteristics and in various frameworks. In this thesis we show the dynamic factor that represents the spatial dynamics in terms of spacetime expansion and contraction in braneworld cosmology framework affects the changes at the throat of the wormhole by either decreasing or increasing the stress energy tensor respectively. This implies an interesting finding concerning the effects of cosmological expansion and contraction of the universe or in general the surrounding space of wormholes that is expanding or contracting from and toward the wormholes respectively. Furthermore, the gravitational lens of a wormhole was also introduced by various researchers. Their treatment was focused basically on the lens signature that describes wormhole geometrical character such as the differences from a black hole or between any various types of wormhole models. The braneworld scenario provides the idea of spacetime with underlying extra-dimensions. The inclusion of extra-dimensional terms in the gravitational lens object spacetime line element will result in some variation in the expression for its gravitational lens deflection angle. In this thesis we investigate such variation by deriving this deflection angle expression. Thus this study not only shows the existence of such variation but also suggests the potential utilization of gravitational lensing to prove the existence of extra dimensions by studying the deflection angle characteristic in accordance with the spacetime expansion rate of the universe.

**Keywords:** General Relativity, Brane, Gravitational Lens, Wormhole.

# ***PENGAJIAN KEATAS DINAMIK RUANG-MASA LOHONG RUANG DALAM MODEL MEMBRANA ALAM***

## **ABSTRAK**

Model-model lohong-ruang dinamik sebelum ini telah diperkenalkan lebih tertumpu kepada kedinamikan lohong-ruang itu sendiri samada perihal lohong-ruang yang berputar atau berevolusi dan di dalam pelbagai kerangka kerja. Dalam tesis ini kami menunjukkan faktor dinamik yang mewakili dinamik ruang dalam terma pengembangan dan pengecutan ruang-masa dalam kerangka kerja kosmologi dunia membrana yang masing-masing memberi kesan kepada perubahan pada rangkungan lohong-ruang samada pengurangan atau penambahan tensor bebanan tenaga. Ini mengimplikasikan suatu penemuan baru yang menarik perihal kesan kosmologi pengembangan dan pengecutan alam atau ruang di sekeliling lohong ruang-lohong ruang yang mengalami pengembangan menjauh dari atau pengecutan kearah lohong ruang tersebut. Selanjutnya, perihal kanta graviti lohong-ruang juga telah diperkenalkan oleh pelbagai penyelidik. Perbincangan tertumpu pada kesan kanta yang memperlihatkan sifat geometri lohong-ruang seperti perbezaan di antara lohong hitam atau di antara pelbagai jenis model-model lohong-ruang. Senario dunia membrana membangkitkan idea ruang-masa yang didasari dimensi-dimensi tambahan. Pemasukan terma dimensi tambahan dalam unsur bentuk ruang-masa objek kanta graviti akan menghasilkan variasi dalam ungkapan sudut lencongan kanta graviti. Maka dalam tesis ini kami telah mengkaji variasi tersebut dengan menerbitkan ungkapan matematik sudut lencongan, maka dalam penyelidikan ini bukan sahaja membuktikan kewujudan variasi ungkapan tersebut malah telah mencadangkan potensi penggunaan kanta graviti dalam pembuktian kewujudan dimensi tambahan dengan mengkaji sifat sudut lencongan selaras dengan kadar pengembangan ruang masa alam.

**Kata kunci:** Kerelatifan Umum, Membrana, Kanta Graviti, Lohong Ruang.

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## LIST OF SYMBOLS AND ABBREVIATIONS

$q_{\mu\nu}$	:	Brane metric tensor
$\chi$	:	Brane Newtonian potential
$\Psi$	:	Brane relativistic potential
$P$	:	Braneworld wormholes radii
$\sigma$	:	Bulk cosmological constant
$y, Y$	:	Bulk-protruding coordinate, hypersurface coordinate
$E_{\mu\nu}$	:	Bulk traceless entity
$\Gamma_{\mu\nu}^{\alpha}$	:	Christoffel symbol or affine connection
$\Lambda, \lambda$	:	Cosmological constant, vacuum energy
$D_{\mu}$	:	Covariant differentiation
$\partial_{\nu}$	:	Differentiation with respect to $x^{\nu}$
$\Theta_{\mu\nu}$	:	Dynamics brane Einstein tensor term
$\xi_{\mu\nu}$	:	Dynamics brane Ricci tensor term
$G_{\mu\nu}$	:	Einstein tensor
$\zeta_{\rho}, \zeta_{p_{rad}}, \zeta_{p_{\perp}}$	:	Energy momentum tensor components dynamic factors
$T^{\mu\nu}, \tau_{\mu\nu}$	:	Energy momentum tensor or stress energy tensor
$\varsigma$	:	<i>Exoticity</i> of exotic matter
$\bar{r}_*$	:	Finite radial distance
$\kappa_5^2$	:	Five Dimensional bulk energy momentum tensor constant
$\forall$	:	For all
$V^{\mu}$	:	Four vector

$u_\mu$	:	Four-velocity vector field of flow
$G$	:	Gravitational constant
$\alpha$	:	Gravitational lens deflection angle
$\xi$	:	Gravitational lens impact parameter
$G_n$	:	Higher dimensional gravitational constant
$s$	:	Interval between any two events
$K_{\mu\nu}$	:	Intrinsic curvature
$K^\mu$	:	Killing vector field
$g_{\mu\nu}$	:	Metric tensor
$G^\mu_\nu$	:	Mixed Einstein tensor
n D	:	n dimensional
$\Lambda_n$	:	n dimensional brane cosmological constant
$^{(n)}g_{\mu\nu}$	:	n dimensional metric tensor
$^{(n)}R^\alpha_{\mu\beta\nu}$	:	n dimensional Riemann curvature tensor
$n_\mu$	:	normal tensor (or vector; tensor of rank 1)
$r_0$	:	Radial distance at the wormhole throat
$\Phi$	:	Red shift function ( $\Phi = \Phi(r)$ )
$R_{\mu\nu}$	:	Ricci tensor
$R^\alpha_{\mu\beta\nu}$	:	Riemann curvature tensor
$r_s$	:	Schwarzschild radius
$b$	:	Shape function ( $b = b(r)$ )
$S_{\mu\nu}$	:	Stress energy tensor of the <i>normal matter</i> in 4 D spacetime
$c$	:	The speed of light

$\exists$	:	There exist
$M$	:	The redshift term of brane metric
$N$	:	The shape function term of brane
$Q$	:	The shape function term of bulk
$a$	:	The surrounding spatial dynamics factor (evolution factor)
$I(\phi)$	:	Wormhole gravitational lens axial angle function
$u$	:	Wormhole gravitational lens impact parameter
$x^\mu$	:	$x^0 = t$ , $x^1 = r$ , $x^2 = \theta$ , $x^3 = \varphi$ of 3 + 1 (4-D) brane
ADD	:	Arkani-Hamed, Dimopoulos and Dvali
ANEC	:	Averaged Null Energy Condition
ASEC	:	Averaged Strong Energy Condition
AWEC	:	Averaged Weak Energy Condition
DEC	:	Dominant Energy Condition
DGP	:	Dvali-Gabadadze-Porrati
ER	:	Einstein-Rosen
EPR	:	Einstein-Podolsky-Rosen
GR	:	General Relativity
KK	:	Kaluza-Klien
NEC	:	Null Energy Condition
RS	:	Randall-Sundram
SEC	:	Strong Energy Condition
WEC	:	Weak Energy Condition

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